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December 1979

EMITTANCE AND TRANSPORT OF ELECTRON BEAMS IN A FREE ELECTRON LASER

By: V. K. Neil

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In this work we examine the consequences of finite beam emittance and discuss some of the requirements on the beam transport system in a free electron laser; we will not discuss the operation of the FEL. We concentrate on beams from linear accelerators, but the transport theory is quite general and may be applied to beams from (or in) a storage ring. We discuss a fundamental limitation placed on the beam current density by the finite emittance and the resulting spread in axial velocity, continuous solenoidal focusing, and the possibility of focusing the electron beam with the magnetic field of the wiggler.

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I INTRODUCTION

In this work we examine the consequences of finite beam emittance and discuss some of the requirements on the beam transport system in a free electron laser. We will not discuss the operation of the FEL, as an extensive theory of the device is presented in a companion paper.[1] We concentrate on beams from linear accelerators, but the transport theory is quite general and may be applied to beams from (or in) a storage ring. The only original work is contained in the final section, which treats focusing by shaping the magnetic field in a planar wiggler.

In Section II we discuss a fundamental limitation placed on the beam current density by the finite emittance and the resulting spread in axial velocity. Section III is devoted to continuous solenoidal focusing. The treatment is based on the beam envelope equation. A derivation of this equation may be found in Ref. 2. The units used in this work are lengths in cm, magnetic fields in kG, and emittance in cm-rad. All currents are in kA except where clearly stated as A.

We examine the possibility of focusing the electron beam with the magnetic field of the wiggler in Sec. IV, and show a simple wiggler magnet design that demonstrates the concept. The treatment employs the theory of Courant and Snyder[3].

For a beam in vacuum with a radius R , emittance ε and current I , the envelope equation is

$$\frac{d^2R}{dz^2} + K^2(z)R = \frac{2I(kA)}{17R(\beta\gamma)^3} + \frac{\varepsilon^2}{R^3} \,. \tag{1.1}$$

In this equation γ is the energy of the particles in units of the rest energy and $\gamma^2 = (1 - \beta^2)^{-2}$. The quantity $K^2(z)$ characterizes the external focusing forces. In Ref. 2, R is the root mean square radius of the beam and ε has a precise mathematical definition. In this work we simply regard R and ε as measurable quantities.

Depending on the values of the parameters $I, \gamma, \varepsilon,$ and R , there are two extreme regions of interest. One of these is known as the "space-charge" dominated region. It should be known as the "self-force" region or some such, but in accelerator jargon all coherent electromagnetic self-forces tend to be lumped into "space-charge". In this region $\varepsilon^2 << 2IR^2/17(\gamma\beta)^3$ so that the second term on the right hand side of Eq. (1.1) may be neglected. The other region is called the emittance-dominated region. In this region the first term on the right hand side of Eq. (1.1) is neglected. For a given ε, I, and γ, the ratio of the two terms is determined by the beam radius. We shall see that for current densities of interest in operation of an FEL, the beam is quite generally in the emittance-dominated region.

II EMITTANCE AND AXIAL VELOCITY SPREAD

The term "emittance" has its roots in beam transport theory. In general, it is defined for each transverse direction (x and y), and may be different in the two directions. The particles in the beam lie within a four dimensional volume in $x, y, dx/dz$, and dy/dz. If the distribution is integrated over y and dy/dz, we are left with a distribution in x and dx/dz. The elipse enclosing the distribution has area equal to $\pi\epsilon_x$. Measured values are quoted as the fraction of the beam that lies within a given area, such as "90% of the beam particles are enclosed in 30π mrad-cm."

In an "ideal" accelerator the transverse forces on the particles are linear in the transverse direction. These forces include those focusing and accelerating the particles. The area in x-P_x (or y-P_y) phase space remains a constant while the longitudinal momentum P_z increases as $\gamma\beta$. Thus dx/dz (or dy/dz) and the emittance decreases as $(\gamma\beta)^{-1}$. It is common practice to introduce the normalized emittance $\epsilon_o \equiv \gamma\beta\epsilon$. In an ideal accelerator ϵ_o is constant throughout the acceleration process. No real accelerator can meet the criterion of linear transverse forces. In addition, the transverse forces must include those arising from the particles' coherent electromagnetic self-fields, which are not linear. It should be noted that the dependence of ϵ on energy in a storage ring does not follow the $(\gamma\beta)^{-1}$ variation because of the effects of synchrotron radiation. If the beam is azimuthally symmetric we have $\varepsilon_x = \varepsilon_y = \varepsilon$, the quantity that appears in Eq. (1.1).

When we consider the emittance of beams from existing rf linacs as well as induction linacs, we discover a serious limitation on the current density of beams from the devices when employed in an FEL. We first consider a beam in which all particles are travelling in the z direction with no transverse velocity component. It is the axial speed that determines whether or not a particle can be trapped in stable phase and radiate coherently in an FEL. If the particle has no transverse velocity but has a deviation $\delta\gamma$ from the value γ_r of the resonant particle, it has a deviation δv_z in axial speed given by

$$\frac{\delta\gamma}{\gamma_r} = \gamma_r^2 \frac{v\delta v_z}{c^2} = \gamma_r^2 \beta \frac{\delta v_z}{c} . \tag{2.1}$$

On the other hand, if a particle has the proper value of γ, but has a transverse speed v_x (or v_y) and total speed v, that particle has a deviation in axial speed given by

$$\delta v_z/v = v_x^2/2v^2 . \tag{2.2}$$

For a beam with emittance ε_x, the maximum value of v_x/v is ε_x divided by the maximum value of x. For azimuthally symmetric beams, the total spread Δv_z in axial speed is given by

$$\Delta v_z/c = \beta\varepsilon^2/2R^2 . \tag{2.3}$$

We compare a monoenergetic beam with an emittance ϵ with a zero emittance beam that has an energy spread $\Delta\gamma$ to define a relation

$$(\gamma\beta\epsilon/R)^2 = (2\Delta\gamma/\gamma)\ \text{equiv}\ , \tag{2.4}$$

meaning that the monoenergetic beam has a Δv_z equivalent to that of the cold beam with energy spread $\Delta\gamma$. A more precise relation is found by including the transverse speed in the equations of motion of particles in an FEL. This relation is

$$(\gamma\beta\epsilon/\mu R)^2 = 2(\Delta\gamma/\gamma)\ \text{equiv}\ , \tag{2.5}$$

in which the quantity μ has the definition

$$\mu^2 = 1 + \left(\frac{e\lambda_w B_w}{2\pi mc^2}\right)^2 . \tag{2.6}$$

In Eq. (2.6) e is the electron charge, m the rest mass, B_w and λ_w the magnitude and wave length of the wiggler magnetic field. Equation (2.6) is valid for a helical wiggler. For a planar wiggler B_w is the root mean square value. Values of μ^2 are typically 1 to 2.

We point out that the effective $\Delta\gamma/\gamma$ in the device arises from three sources, and may be expressed in the form

$$\left(\frac{\Delta\gamma}{\gamma}\right)_{eff} = \left(\frac{\Delta\gamma}{\gamma}\right)_{act} + \left(\frac{\Delta\gamma}{\gamma}\right)_{\epsilon} + \left(\frac{\Delta\gamma}{\gamma}\right)_{w} ,$$

in which the first term on the right hand side denotes the actual spread in energy in the beam and the second term is expressed by Eq. (2.5). The third term on the right hand side denotes the change in axial velocity arising from the transverse variation in the wiggler field, and will depend on the particular wiggler configuration. In this work we deal only with the contribution from the emittance, and $\Delta\gamma/\gamma$ in this section and in Section III refers to this contribution. The contribution from variations in the wiggler may be comparable to that from the emittance and is discussed briefly in Section IV.

For a large number of existing rf linacs, there is an empirical relation between the measured values of ϵ and the average beam current I. The relation is

$$\epsilon_o = \gamma\beta\epsilon = 0.3\ I^{1/2}(\mathrm{kA})\ \mathrm{cm\text{-}rad}. \tag{2.7}$$

with I the <u>time average</u> current during the macropulse from the accelerator.

This equation means that the output values of I, γ, and ϵ for any individual accelerator are related in this manner. The coefficient varies by a factor of 2 or 3 among rf linacs. Many rf linacs have several different modes of operation. The average current and the energy are different in different modes. Generally Eq. (2.7) is obeyed for any mode, but again the coefficient may vary by a factor of 2 or 3. The value of 0.3 in Eq. (2.7) represents a lower limit for rf linacs currently in operation.

The average current out of rf linacs is typically 10's of mA to as much as a few A. But Eq. (2.7) comes close to fitting measured values from the Astron linear induction accelerator. The current was constant over the pulse duration of about 250 ns. Measurements of the emittance of the beam from that device at $\gamma = 11$ and $I = 300$ A to 500 A indicate that, for this device, the coefficient 0.3 is a factor of 2 too large.

A possible explanation of the validity of Eq. (2.7) over 5 orders of magnitude in current may be stated as follows: all injectors, sources, or guns used in electron linacs inject approximately the same density of particles into the four dimensional phase space x,y,P_x,P_y. The volume is thus proportional to the total current I, and the area in x-P_x or y-P_y phase space is proportional to $I^{1/2}$.

We note that the total current I is approximately related to the current density J by

$$I = \pi R^2 J \ . \tag{2.8}$$

We use this relation in Eq. (2.7) to obtain

$$(\gamma\beta\epsilon/R)^2 = 0.09 * J\ (kA/cm^2) \qquad (2.9)$$

By using Eq. (2.5) we can now determine what time average current density J_c corresponds to an axial velocity spread equivalent to a fractional energy spread of 1%. For $\mu^2 = 2$ we find

$$J_c = 140\ A/cm^2 . \qquad (2.10)$$

For rf linacs the peak current may be orders of magnitude greater than the average current.

There is nothing unique about the value of 1% for $\Delta\gamma/\gamma$, it is merely illustrative. The input laser power per unit area necessary to trap particles in stable phase varies as $(\Delta\gamma)^4$, and apparently varies as ϵ^8 if the transverse velocity spread is the major contributor to the spread in axial speed. Clearly lowering the emittance by a factor of 2 or so would be desirable. But in order to achieve average current densities of several kA/cm^2, we must have an emittance an order of magnitude lower than typically achieved to date.

One further observation is in order with regard to the current density limitations. The value of J_c is independent of the beam energy if $\gamma\epsilon$ is really a constant. As pointed out earlier, in any given accelerator this is an ideal situation not likely to be achieved, so that the value of J_c may well decrease with γ. On the other hand, the value of the actual energy spread $\Delta\gamma$ can reasonably be expected to remain rather constant, so that $\Delta\gamma/\gamma$ decreases with energy.

III SOLENOIDAL TRANSPORT

We first consider transport in a continuous axial magnetic field B that is radially uniform. In this transport system the quantity K^2 in Eq. (1.1) is independent of z and given by

$$K^2 = (2\rho)^{-2} , \tag{3.1}$$

in which ρ is the radius of gyration the beam particles would have in the field B if their motion were entirely perpendicular to the field. In Gaussian units, we have

$$\rho = \frac{pc}{eB} , \tag{3.2}$$

As an engineering formula, we use

$$\rho(\mathrm{cm}) = 1.7\ \gamma\beta/B(\mathrm{kG}) . \tag{3.3}$$

The equilibrium, or "matched" radius of the beam is found by setting $d^2R/dz^2 = 0$. We introduce the quantities R_s and R_ε by the definitions (lengths in cm, I in kA , ε in cm-rad):

$$R_s^2 = 8I\rho^2/17(\gamma\beta)^3, \tag{3.4}$$

$$R_\varepsilon^2 = 2\rho\varepsilon \ . \qquad (3.5)$$

Physically, R_s is the matched radius of a "zero emittance" beam and R_ε in the matched radius of a "low current" beam (i.e., a beam with finite emittance and current I sufficiently small that it is in the emittance dominated region). Since R_ε varies as $B^{-1/2}$ while R_s varies as B^{-1} , for any values of $I, \gamma,$ and ε the ratio of these two quantities may be adjusted by changing the value of B . In terms of R_s and R_ε , the matched radius R_m is given by

$$R_m^2 = (R_s^2/2) + [(R_s^4/4) + R_\varepsilon^4]^{1/2} \ . \qquad (3.6)$$

If the emittance of the beam is zero, motion of particles in the beam is laminar. The terms "laminar flow" and "Brillouin" flow are used to describe this motion. The simple treatment here does not describe exactly the condition for laminar flow for relativistic particles. A thorough treatment of a zero emittance relativistic beam in a uniform axial magnetic field has been done by Reiser[4]. The results of Ref. 4 show that all particles in the beam have the same axial speed. Particles remain at a constant radius and execute helical orbits, but the azimuthal velocity varies with r . The particles' kinetic energy as well as the charge density ρ , axial and azimuthal current densities j_z and j_θ , and field components E_r , B_θ , and B_z all are functions of radius in Reiser's theory. But the axial speed is independent of radius.

The variation of kinetic energy (i.e. variation of γ) arises from the electrostatic potential of the charge distribution in the beam. For a beam with radially uniform charge density and $v_z = c$, the difference in potential energy between the axis of the beam and the edge of the beam is 30 kV per kA of beam. In laminar flow the difference in kinetic energy is manifest in radially varying azimuthal speed, while the axial speed remains constant. This ideal model can never be realized in practice because all real beams have a finite emittance. If the magnetic field is adjusted so that $R_\epsilon^2 << R_s^2$, (i.e., transport in the space-charge dominated regime), the condition of Brillouin flow can perhaps be approximately achieved. Under such circumstances it might be reasonable to assume that the change in potential across the beam leads to a negligible spread in axial velocity. The actual situation would depend on the details of the electron distribution in six dimensional phase space (or 5 dimensional if the beam is indeed azimuthally symmetric). But even in the space-charge dominated regime, a finite emittance gives a spread in axial velocity according to the relation (2.5). If the empirical formula (2.7) holds, then Eqs. (2.9 and 2.10) are still valid. By transporting the beam in the space-charge dominated regime the axial velocity spread from the potential drop across the beam may be reduced to a negligible value, but the velocity spread from the emittance still leads to a maximum current for an equivalent $\Delta\gamma/\gamma$, as expressed in Eq. (2.10) for an equivalent $\Delta\gamma/\gamma$ of 10^{-2}.

We will now present some examples of beam radii, solenoidal magnetic field amplitudes, and beam currents. In the first examples we assume that the emittance is given by Eq. (2.7), and we are considering an induction linac or any other accelerator that produces constant current during the pulse. We set $d^2R/dz^2 = 0$ in Eq. (1.1) and employ Eqs. (2.7, 3.1, and 3.3.) to obtain

$$B^2R_m^2 = 11.56\ I(kA)\ [(2/17\gamma\beta) + (0.09/R_m^2)]\ . \qquad (3.7)$$

If we use Eqs. (2.5 and 2.9) with $\mu = 2$ we can express I in terms of the equivalent $\Delta\gamma/\gamma$ allowed. We have

$$I(kA) = 44\ (\Delta\gamma/\gamma)\ R_m^2\ , \qquad (3.8)$$

and Eq. (3.7) becomes

$$B^2 = 508\ (\Delta\gamma/\gamma)\ [(2/17\gamma\beta) + (0.09/R_m^2)]. \qquad (3.9)$$

We may calculate the ratio $(R_\varepsilon/R_s)^2$ from Eqs. (2.7, 3.1, 3.4 and 3.5). We obtain

$$(R_\varepsilon/R_s)^2 = 3\gamma\beta B/4I^{1/2}\ . \qquad (3.10)$$

This ratio is a measure of the extent to which the beam is being transported in the space-charge dominated regime. (Small values indicate space-charge regime, large values indicate emittance regime.) Values of this ratio along with values of I, R_m, and B are given in Table 1. for $\gamma = 10$ and $\Delta\gamma/\gamma = 10^{-2}$. From the values of $(R_\varepsilon/R_s)^2$ we see that the beam is in the emittance dominated regime even for this low value of γ .

Any value of γ higher than this will result only slightly smaller values of B, since the first term on the right hand side of Eq. (3.9) is much smaller than the second term for $\gamma = 10$ and decreases with increasing values of γ.

In the emittance dominated regime the $\Delta\gamma/\gamma$ arising from the potential drop across the beam must be considered, but it amounts to only 3×10^{-3} for $I = 440A$.

TABLE I

Values of I, B, R_m and $(R_\varepsilon/R_s)^2$ calculated from Eqs. (3.9 and 3.10) for $\gamma = 10$, $\Delta\gamma/\gamma = 10^{-2}$. Note that I is in amperes.

R_m(cm)	I(A)	B(kG)	$(R_\varepsilon/R_s)^2$
0.2	17.6	3.4	190.
0.5	110.	1.37	30.
1.0	440.	.72	8.0

We now repeat the above calculation, but reduce the emittance by order of magnitude. We assume that ε still varies as $I^{1/2}$, but change the coefficient in Eq. (2.5) from 0.3 to 3×10^{-2}. The equation analogous to Eq. (3.8) is now

$$I = 4.4 \times 10^3 \, (\Delta\gamma/\gamma) \, R_m^2 \, . \qquad (3.11)$$

and Eq. (3.9) becomes

$$B^2 = 5.08 \times 10^4 \, (\Delta\gamma/\gamma) \, [(2/17\gamma\beta) + (9 \times 10^{-4}/R_m^2)] \, , \qquad (3.12)$$

while $(R_\varepsilon/R_s)^2$ is now given by

$$(R_\varepsilon/R_s)^2 = 0.3\gamma\beta B/4I^{1/2} \, . \qquad (3.13)$$

Values of this ratio as well as I, B, and R_m are shown in Table 2 for $\Delta\gamma/\gamma = 10^{-2}$ and $\gamma = 10$ and 100. The required magnetic fields are rather substantial. For $R_m \leq 0.5$ cm , the values of B lie within a factor of 2 of each other for $\gamma = 100$ and $\gamma = 10$, the latter again being taken as a lower extreme.

We now apply Eqs. (3.12 and 3.13) to a low energy beam with $\gamma\beta = 2$ corresponding to a kinetic energy of 630 kev. An FEL employing such a lower energy beam will require an electromagnetic pump (wiggler) and the allowable energy spread will be much lower than that for a device employing a fixed magnetic field wiggler. Results are shown in Table 3 for $\Delta\gamma/\gamma = 10^{-4}$. Only at very small radii is the beam in the emittance-dominated regime, but $\Delta\gamma/\gamma$ from the potential drop is negligible for the allowed current levels. At $R_m = 0.5$ cm and 1.0 cm the beam is in the space-charge dominated regime, and if the $\Delta\gamma/\gamma$ from the potential drop is not reduced, it is much larger than the 10^{-4} allowed.

TABLE 2

Values of I, B, R_m and $(R_\varepsilon/R_s)^2$ calculated from Eqs. (3.12 and 3.13) for $\Delta\gamma/\gamma = 10^{-2}$, $\gamma = 10$ and 100.

R_m(cm)	I(kA)	B(kG)		$(R_\varepsilon/R_s)^2$	
		$\gamma = 10$	$\gamma = 100$	$\gamma = 10$	$\gamma = 100$
0.1	.44	7.2	6.8	8.	75.
0.2	1.76	4.2	3.5	2.4	20.
0.5	11.	2.8	1.6	0.6	3.6
1.0	44.	2.5	1.0	0.3	1.1

TABLE 3

Values of I, B, R_m and $(R_\varepsilon/R_s)^2$ calculated from Eqs. (3.12 and 3.13) for $\gamma = 2$ and $\Delta\gamma/\gamma = 10^{-4}$. Note that I is in amperes.

R_m(cm)	I(A)	B(kG)	$(R_\varepsilon/R_s)^2$
0.05	1.1	1.46	6.6
0.1	4.4	.87	1.9
0.2	17.6	.64	.7
0.5	110	.56	.25
1.0	440	.55	.12

We conclude that even with an emittance an order of magnitude lower than that given by Eq. (2.5) the current in a low energy FEL is seriously limited.

As a final example we consider an rf linac. Equation (2.5) gives the emittance in terms of the time average current, but the first term on the right hand side of Eq. (1.1) contains the instantaneous, or peak, current in a micropulse. Equation (3.7) must be modified to take this into account. We have

$$B^2 R_m^2 = 11.56\ I_{ave} \left[(2\ I_{peak} / 17\ \gamma\ I_{ave}) + (0.09/R_m^2) \right] . \quad (3.14)$$

For $R_m = 0.1$ cm , $I_{ave} = 4.4$ A is required to create a current density of 140 A/cm^2 . We take $I_{ave} = 2$A , $I_{peak} = 20$ A , $R_m = 0.1$ cm, $\gamma = 40$, and obtain $B = 4.6$ kG . From Eqs. (2.5 and 2.7) we have an equivalent $\Delta\gamma/\gamma$ of 4×10^{-3} , which is probably less than the actual $\Delta\gamma/\gamma$ in such a device. So in this example, at least, the emittance is not the major contributor to the axial velocity spread.

IV FREE DRIFT AND WIGGLER FOCUSING

As we have seen in the previous section the solenoidal magnetic fields necessary to transport the electron beam are generally a few kG. Although such fields can certainly be achieved, experimental hardware would be more manageable if there were no solenoid surrounding the wiggler. In fact, a solenoidal field cannot be employed to transport the beam if the wiggler consists of iron magnets. Let us first consider the consequences of focusing the beam at the entrance to the wiggler so that a beam waist occurs somewhere near the middle of the wiggler (or the interaction region for an electromagnetic wiggler). The focusing could be accomplished with a quadruple doublet or triplet.

If the beam is in the emittance-dominated regime, we may neglect the first term on the right hand side of Eq. (1.1). In the drift region $K^2(z) = 0$ and Eq. (1.1) is easily integrated. If we measure the axial position z from the beam waist where the radius is R_w, we obtain

$$R^2(z) = R_w^2 + (\epsilon z/R_w)^2 \ . \tag{4.1}$$

The behavior of the electron beam is the same as that for the laser beam near the focus. The area of the beam doubles at a distance L from the waist, with L given by

$$L = R_w^2/\epsilon \ . \tag{4.2}$$

For a numerical example we use the "improved" emittance $\gamma\epsilon = 0.03\ I^{1/2}$. We set R_w equal to R_m and take the values of R_m and I from Table 2. For $R_w = 0.2$ cm and $I = 1.76$ kA we find $L = \gamma$ cm . For $R = 0.5$ cm and $I = 11$ kA , we find $L = 2.5\ \gamma$ cm .

The numbers indicate that the electron beam cannot be cast more than a few meters. It might be possible to interrupt a magnetic wiggler and insert additional focusing elements, but this must be done carefully in order to preserve the phase of the electron beam with respect to the pondermotive wave. In an FEL employing an electromagnetic wiggler, the beam can be periodically focused with little difficulty.

Let us now examine the possibility of focusing the beam with the wiggler itself. This concept was suggested by Phillips[5]. In this discussion we will merely give one example of a magnet configuration that provides equal focusing in both transverse planes. The configuration is in no way optimized in the sense of reducing the axial velocity spread arising from finite emittance and energy spread.

In the following treatment we use the work of Ref. 3. We first point out that a "square edge" wiggler as shown in Fig. 1 provides focusing in the y direction but not in the x direction. As defined in Fig. 1, the magnetic field is alternately in the $\pm y$ direction causing particles to oscillate in the x direction. Particles cross the edges at an angle,

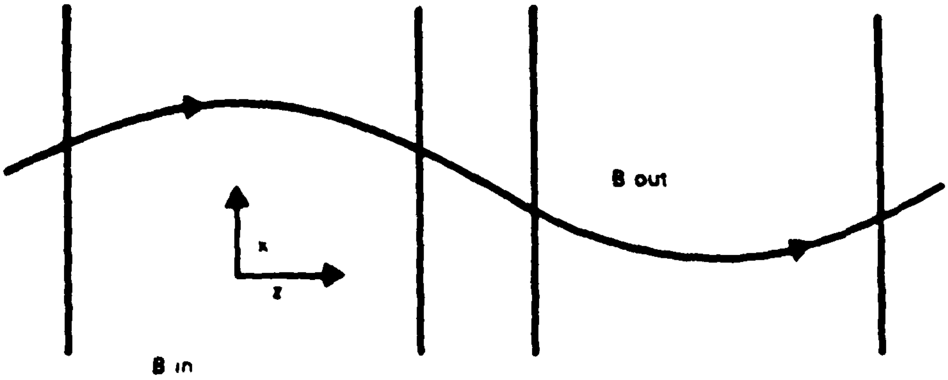

FIGURE 1

ORBIT OF REFERENCE PARTICLE IN A SQUARE-EDGE WIGGLER
WITH UNIFORM MAGNETIC FIELD.

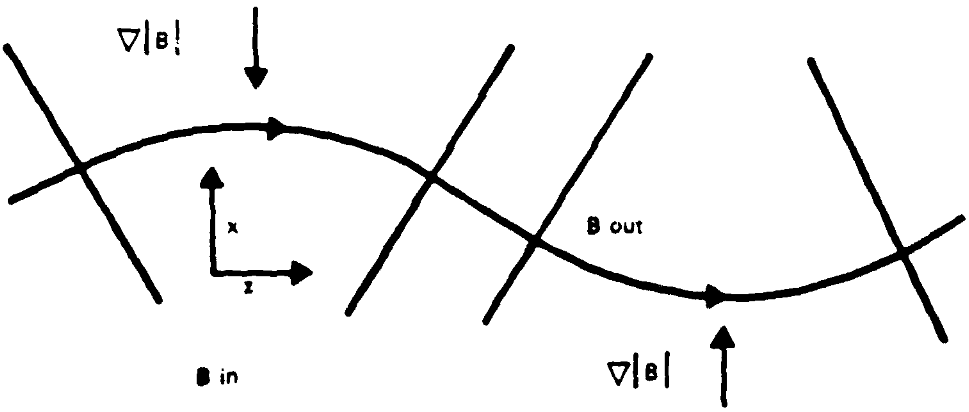

FIGURE 2

ORBIT OF REFERENCE PARTICLE IN A NORMAL EDGE WIGGLER
WITH $d|B|/dr < 0$.

and if they are off the median plane $\left(|y| > 0\right)$ they encounter a component of magnetic field B_z near the edge, and the $v_x B_z$ force is focusing in the y direction. The edge effect is defocusing in the x direction, but the total focusing across the magnet is zero.

Now consider the magnet configuration shown in Fig. 2. The particles now cross the edges at a right angle, so that there is no focusing or defocusing at the edges. The magnetic field in the magnet is not uniform, but decreases with radius as shown in Fig. 2. For those familiar with the terms, this is an $n = 1/2$ weak focusing bending magnet. We define a reference orbit going through the magnets. This particle crosses the edge at a right angle at the proper orbit radius $r = \rho$ for its energy. This is the particle for which the magnet is designed--it is the "resonant" particle of the FEL. For the present we will neglect energy variations in the beam. Other particles near the reference orbit are at a radius $r = \rho + x$. The equation of motion for these particles is

$$\frac{d^2x}{ds^2} + \left(\frac{\nu_x}{\rho}\right)^2 x = 0 \quad , \tag{4.3}$$

in which s is defined as the distance along the reference orbit, and ν_x^2 is defined by the relation

$$\nu_x^2 = 1 + n \quad , \tag{4.4}$$

where n is the field index defined as

$$n = -\left(\frac{\rho}{B}\,\frac{dB}{dr}\right)_{r=\rho} \quad . \tag{4.5}$$

For a weak focusing bending magnet, dB/dr is negative and n is positive.

The equation of motion in the y direction is

$$\frac{d^2y}{ds^2} + \left(\frac{\nu_y}{\rho}\right)^2 y = 0 \quad , \tag{4.6}$$

in which $\nu_y^2 = -n$. So if we make $n = 1/2$, the focusing is the same in both transverse directions. It is not at all clear that equal focusing in the two planes is desirable in practice. For a planar wiggler it might be desirable to make the emittance in $y - y'$ less than in $x - x'$ so that less focusing is required for y motion. But for simplicity here we choose $\nu_x = \nu_y = 2^{-1/2}$, so that we need consider motion in one direction.

If a particle has x_o and x_o' upon entering the first magnet, it has x and x′ leaving the first magnet and the values are related by the matrix equation

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{vmatrix} \cos\psi & (\rho/\nu)\sin\psi \\ -(\nu/\rho)\sin\psi & \cos\psi \end{vmatrix} \begin{pmatrix} x_o \\ x_o' \end{pmatrix} \quad . \tag{4.7}$$

Inthisrelationtheangle $\psi = \nu\theta$, where θ isthetotalbendingangle in the magnet. Although it is obvious that the same relation holds for y motion in the second magnet, it takes a little thought to convince oneself that it also holds for z motion. By definition, both x and x' change sign in the second magnet, but both B and dB/dx also change sign.

A drift length d is shown in Fig. 2. The transformation matrix for a drift is

$$M_d + \begin{vmatrix} 1 & d \\ 0 & 1 \end{vmatrix} \quad . \tag{4.8}$$

Including the drift region in our calculation would alter the results very little if $d \ll \rho\theta$, so we will neglect it for simplicity.

Since focusing is the same in both planes, the matched beam is round and has a radius given by

$$R = \left(\rho\epsilon/\nu\right)^{1/2} = \left(\sqrt{2}\ \rho\epsilon\right)^{1/2} \quad . \tag{4.9}$$

(If $\nu_x \neq \nu_y$, the maximum extent of the beam in x and y are $\left(\rho\epsilon_x/\nu_x\right)^{1/2}$ and $\left(\rho\epsilon_y/\nu_y\right)^{1/2}$ respectively.) From Eq. (3.2) we have

$$\sqrt{2}\ \rho = \sqrt{2}\ \frac{\gamma\beta mc^2}{eB} = \frac{\gamma\beta\lambda_w}{2\pi} \left(\sqrt{2}\ \frac{mc^2}{eB}\ \frac{2\pi}{\lambda_w}\right) \quad . \tag{4.10}$$

The quantity in parentheses is the same as that occurring in Eq. (2.6), and for operation of an FEL is unity or close to it, so that Eq. (4.9) becomes

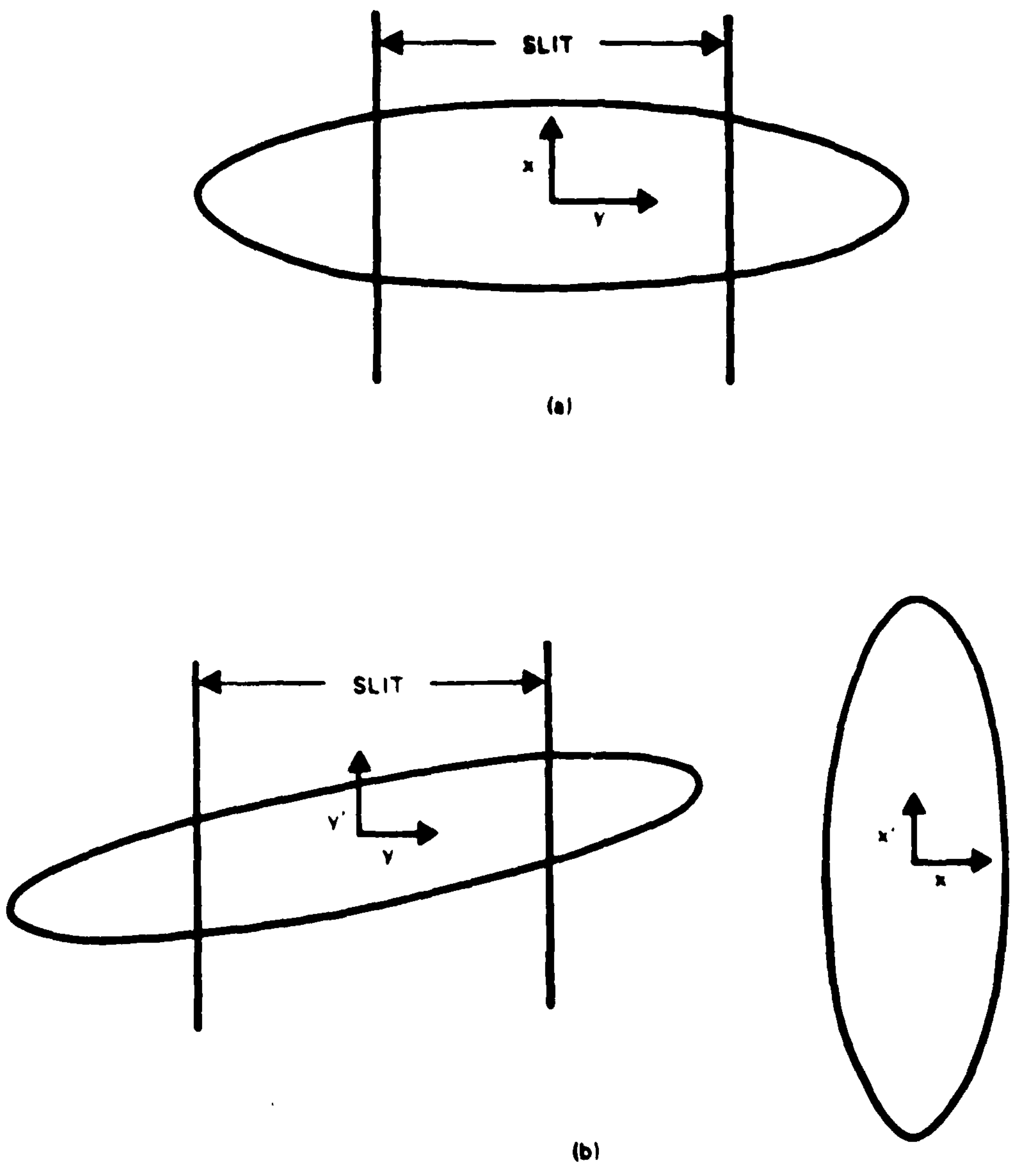

FIGURE 3

CROSS-SECTION OF BEAM AT FOCUS IN x PLANE AFTER PASSING THROUGH QUADRUPOLE (a), AND PHASE ELIPSES AT FOCUS IN x (b).

$$R^2 = \lambda_w \gamma\beta\epsilon/2\pi \quad ,$$

or

$$\gamma\beta\epsilon/R = 2\pi R/\lambda_w \quad . \tag{4.11}$$

The left hand side of Eq. (4.11) is related to the equivalent $\Delta\gamma/\lambda$ treated in Section II. Employing Eq. (2.5) with $\mu^2 = 2$ in Eq. (4.11) we find

$$\left(\Delta\gamma/\gamma\right)_\epsilon^{1/2} = \pi R/\lambda_w \quad . \tag{4.12}$$

If we choose R/λ_w satisfying this relation (e.g. $\Delta\gamma/\gamma = 10^{-2}$, $\pi R/\lambda_w = 0.1$ then the weak focusing wiggler magnets treated here will transport the allowable current. For R/λ_w greater than that necessary to satisfy Eq. (4.12), the wiggler will transport more than the allowable current. There may be restrictions on the value of R/λ_w arising from the variation of B_y with y . A detailed analysis of the actual field pattern will be necessary to determine this restriction, but apparently even this simple wiggler design provides adequate focusing.

We point out that, if the drift space is included in the calculation, the beam is slightly larger in the center of the magnet and slightly smaller in the center of the drift space.

The discussion in Sec. 2 leads to a relation between the current density and the maximum allowable $\Delta\gamma/\gamma$. From Eqs. (2.5) and (4.11) we can derive a relation between the beam current I and λ_w . We find

$$KI^{1/2} = \frac{2\lambda_w}{\gamma} \left(\frac{\Delta\lambda}{\gamma}\right)_\varepsilon \quad . \qquad (4.13)$$

in which K has been taken as 3×10^{-2} in this section. The relation holds for any K value and for focusing such that Eq. (4.12) is valid. It is interesting to note that, for $\mu^2 = 2$, the contribution $(\Delta\gamma/\gamma)_w$ from the variations of the wiggler magnetic field in the y direction is given approximately by

$$\left(\frac{\Delta\gamma}{\gamma}\right)_w = \left(\frac{\pi R}{\lambda_w}\right)^2 \quad . \qquad (4.14)$$

Thus Eq. (4.11) shows that $(\Delta\gamma/\gamma)_\varepsilon = (\Delta\gamma/\gamma)_w$, a condition that minimizes the sum of the two contributions.

Further study of wiggler configurations will seek arrangements that reduce the axial velocity spread caused by the emittance. It may be possible to accomplish this in one plane only, preferably the x plane. As mentioned above, ε_y need not be as large as ε_x . A beam in a storage ring has $\varepsilon_y << \varepsilon_x$. In a beam from a linac the emittance in one or both planes may be reduced at the expense of lowering the current. Suppose we make the beam wide in the y direction and narrow in the x direction. The easiest way to do this is to pass the beam through a quadrupole that focuses in x and defocuses in y . The beam cross-section is shown in Fig. 3a, and the two phase elipses in Fig. 3b. In this configuration the beam is placed through a slit as shown, reducing the extent in y and ε_y . If the y elipse is uniformly filled we reduce the total current by the same fraction that ϵ_y is reduced. But generally the phase density is higher near the center of the elipse so that the reduction in current is less. There is little or no reduction in ϵ_x , but the process may be repeated in the x plane if desired. This process would be particularly useful if applied to the beam out of an induction linac, which carries more current than can be used in an FEL.

In conclusion, we see that the solenoidal magnetic field necessary to transport the allowed current in an FEL is at most a few kG. In this section we have an existence proof that the focusing can also be accomplished by shaping the magnetic field of the wiggler.

Although an improvement in emittance is certainly desirable, the results of Section III indicate that values of emittance currently achieved in rf linacs will permit the use of these devices for an FEL used as an oscillator. For an FEL employing a magnetic wiggler and operated as an amplifier, an improvement of a factor of 3 to 10 in the emittance from induction linacs is desirable. For an FEL employing an electromagnetic wiggler, such an improvement is essential. An improvement of two orders of magnitude would make these devices interesting.

We mention that the phase-displacement concept relaxes to some extent the requirement for small fractional energy spread. With this scheme particles are not trapped in stable phase with respect to the ponderomotive wave, and there is no connection between the effective energy spread and the input laser power. A larger energy spread does necessitate a longer wiggler to extract the same energy per particle from all the particles in the beam.

REFERENCES

1. N. M. Kroll, P. Morton and M. N. Rosenbluth, "Free Electron Lasers with Variable Parameter Wigglers", JASON Technical Report JSR-79-01. SRI International (1979).

2. E. P. Lee and R. K. Cooper, "Particle Accelerators", 7, 83, (1976).

3. E. D. Courant and H. S. Snyder, Annals of Physics, 3, 1, (1958).

4. M. Reiser, The Physics of Fluids, 20, 477, (1977).

5. R. M. Phillips, I.R.E. Trans. on Electron Devices, 7, 231, (1960).

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